ME-221
PROBLEM SET 11

Problem 1

The Bode diagram of a second order underdamped system is shown in Figure 1. The system has a complex conjugate pole at $p_{1,2} = -2 \pm j$ and a zero at $z_1 = -1$

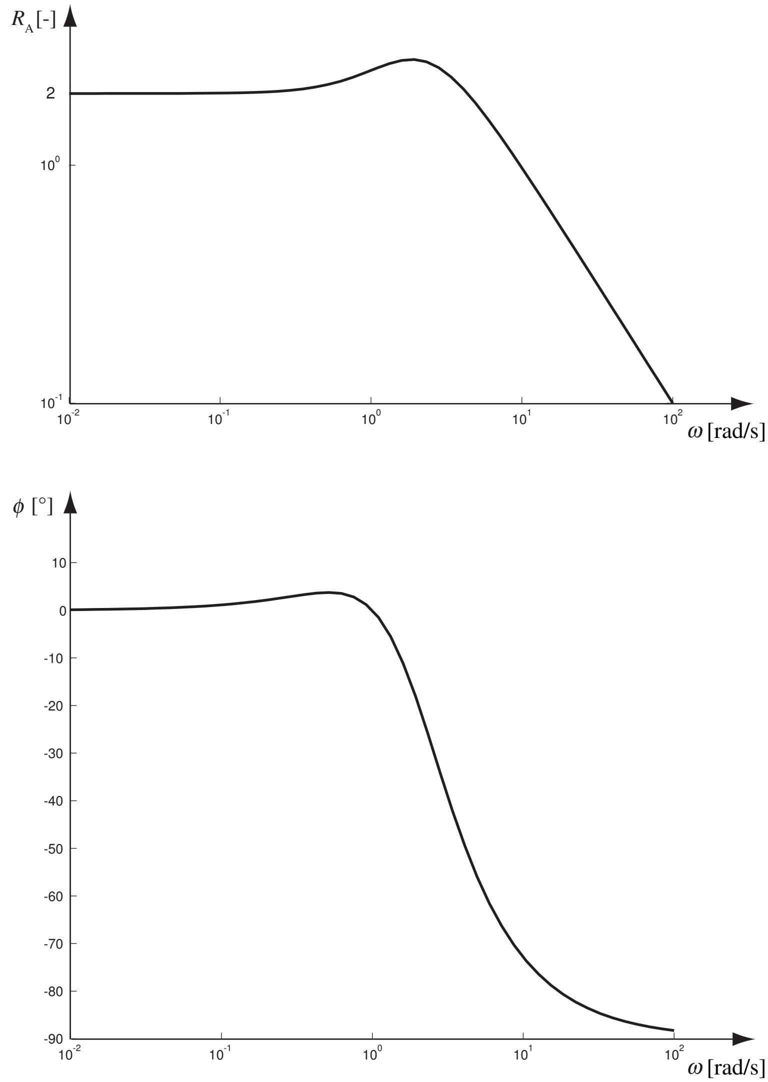

Figure 1: Bode plot

a) Find the transfer function of the system. Calculate the natural frequency of oscillation ω_0 and the damping coefficient ζ of the second order term.

b) Does the system have a resonant peak? If yes, find the resonant frequency and the magnitude of the peak. How would the Bode plot change if we removed the zero from the transfer function?

c) Would you consider this system as a low-pass or high-pass filter?

Problem 2

The unit step response of a second order dynamical system is shown in Figure 2.

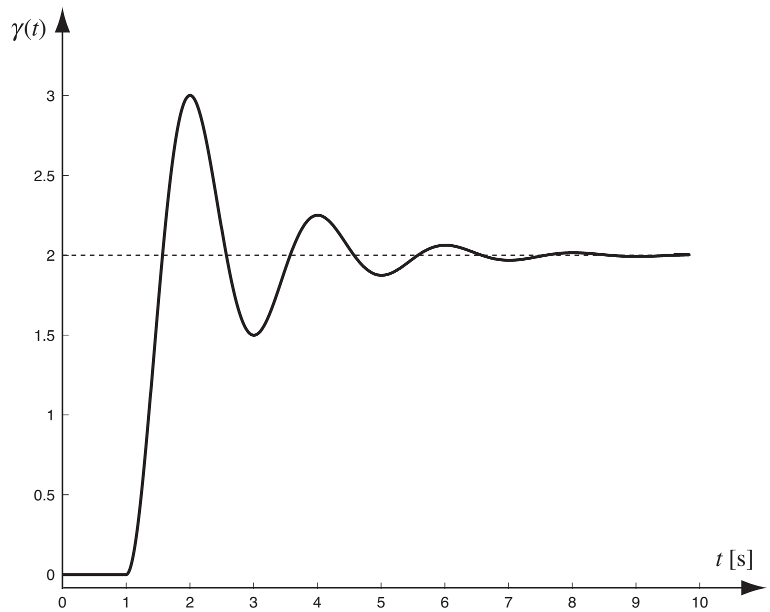

Figure 2: Transient response in the time domain

a) Determine the transfer function of the system given that the system has no zeros.

b) Draw the Bode diagram showing the asymptotes of the magnitude and phase.

c) What would be the effect of decreasing the damping coefficient ζ by half on resonant frequency and magnitude of the resonant peak while keeping the static gain and natural frequency of the system constant? What would be the implications of this change in the time-domain performance metrics (maximum overshoot, settling time, and peak time)?

Problem 3

Sketch the Bode plot (magnitude and phase) of the following systems.

a) $G(s) = \dfrac{2se^{-s}}{s^2 + 3s + 2}$

b) $G(s) = \dfrac{s + 100}{s(s + 20)(s^2 + s + 1)}$

Problem 4

Consider a mechanical system described by the following differential equation. The system is initially at rest.

$$\ddot{y}(t) + \dot{y}(t) + y(t) = 2u(t)$$

a) Find the transfer function $G(s)$ of the system and sketch the Bode plot.

b) We would like to design a first order filter $F(s) = \dfrac{K}{\tau s + 1}$ in a way that the new system with the transfer function $G'(s) = G(s)F(s)$ has magnitude $|G'(j\omega)| = 1$ and phase angle $\phi = -3\pi/4$ at frequency $\omega = 1$.